Special Section (MAT435/MAT235): Common Errors in Mathematics

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This special section covers the errors that the students often make in doing algebra, and not just errors typically made in an algebra class. These mistakes, in fact, make by students at all level of studies. The earlier examples are the errors commonly done in a math class not limited to the calculus class.

Algebra Errors

Case 1. Division by Zero

☑ $\frac{0}{2} = 0$ ☒ $\frac{2}{0} = 0$ or $\frac{2}{0} = 2$

Division by zero is undefined! You simply cannot divide by zero so don’t do it!

Case 2. Bad/lost/Assumed Parenthesis

<u>Example 1</u>. Square $3x$

☑ $(3x)^2 = (3)^2(x)^2 = 9x^2$ ☒ $3x^2$

Parenthesis are required in this case to make sure we square the whole thing, not just the x, so don’t forget them!

<u>Example 2</u>. Square -4

☑ $(-4)^2 = (-4)(-4) = 16$ ☒ $-4^2 = -(4)(4) = -16$

Parenthesis are required in this case especially when you use the calculator.

<u>*Example 3*</u>. Subtract from $3x-5$ from x^2+2x-5

☑ $x^2+2x-5-(3x-5)=x^2-x$ ☒ $x^2+2x-5-3x-5=x^2-x-10$

<u>*Example 4.*</u> Convert $\sqrt{6x}$ to fractional exponents.

☑ $\sqrt{6x}=(6x)^{1/2}$ ☒ $\sqrt{6x}=6x^{1/2}$

<u>*Example 5*</u>. Evaluate $-3\int 6x-2dx$

☑ $-3\int 6x-2dx=-3(3x^2-2x)+c$
$=-9x^2+6x+c$

☒ $-3\int 6x-2dx=-3.3x^2-2x+c$
$=-9x^2-2x+c$

Case 3. Improper Distribution

<u>*Example 1*</u>. Multiply $3(3x^2-9)$

☑ $3(3x^2-9)=9x^2-27$ ☒ $3(3x^2-9)=9x^2-9$

Make sure that you distribute the 3 all the way through the parenthesis! Too often people just multiply the first term by the 3 and ignore the second term.

<u>*Example 2*</u>. Multiply $3(2x-5)^2$

☑ $3(2x-5)^2=3(4x^2-20x+25)$
$=12x^2-60x+125$

☒ $3(2x-5)^2=(6x-15)^2$
$=36x^2-180x+225$

Remember that exponentiation must be performed **BEFORE** you distribute any coefficients through the parenthesis!

Case 4. Additive Assumption

- $(x+y)^2 \neq x^2 + y^2$
- $\frac{1}{(x+y)} \neq \frac{1}{x} + \frac{1}{y}$
- $3(2x-5)^2$
- $(x+y)^2 \neq x^n + y^n$ for any integer $n \geq 2$
- $\sqrt{x^2+y^2} \neq x + y$
- $\sqrt[n]{x+y} \neq \sqrt[n]{x} + \sqrt[n]{y}$ for any integer $n \geq 2$
- $\log \sqrt{x} \neq \sqrt{\log x}$

Case 5. Cancelling Errors

Example 1. Simplify $\frac{2x^4 - x}{x}$.

☑ $\frac{2x^4 - x}{x} = \frac{x(2x^3-1)}{x} = 2x^3 - 1$ ☒ $\frac{2x^4 - x}{x} = 2x^3 - x$ or $2x^4 - 1$

Example 2. Solve $4x^2 - x$.

☑ $4x^2 - x = 0$
$x(4x-1) = 0$
$x = 0, \frac{1}{4}$

☒ $4x = 1$
$x = \frac{1}{4}$

Often, many students get used to just cancelling (i.e. simplifying) things to make life easier. So, the biggest mistake in solving this kind of equation is to cancel an *x* from both sides to get $x = \frac{1}{4}$, while, there is another solution that we've missed, i.e. $x = 0$.

Case 6. Proper Use of Square Root

☑ $\sqrt{16} = 4$ ☒ $\sqrt{16} = \pm 4$

This misconception arises because they are also asked to solve things like $x^2 = 16$. Clearly the answer to this is $x = \pm 4$ and often they will solve by "taking the square root" of both sides. There is a missing step however. Here is the proper solution technique for this problem.

$$x^2 = 16$$
$$x = \pm\sqrt{16} \rightarrow x = \pm 4$$

Case 7. Ambiguous Fractions

<u>Example 1</u>. Writing 2/3*x* in a proper way.

When writing "/" to denote a fraction, it should be made clear, which fraction that this 2/3*x* represent? This can be either of the two following fractions :

$$\frac{2}{3}x \text{ or } \frac{2}{3x}$$

If you intend for the *x* to be in the denominator then write it as such that way, $\frac{2}{3x}$, *i.e.* make sure that you draw the fraction bar over the WHOLE denominator. If you don't intend for it to be in the denominator then don't leave any doubt! Write it as $\frac{2}{3}x$.

<u>Example 2</u>. Writing a + b/c + d in a proper way.

This fraction can be either of the two following fractions : $a + \frac{b}{c} + d$ or $\frac{a+b}{c+d}$

This is definitely NOT the original fraction. So, if you MUST use "/" to denote fractions use parenthesis to make it clear what is the numerator and what is the denominator. So, you should write it as $(a+b)/(c+d)$.

Trigonometric Errors

Case 1. Degree vs Radian

$$\sin(10) = 0.173648178 \quad \text{(in degree)}$$
$$\sin(10) = -0.54421111 \quad \text{(in radian)}$$

So, be careful and make sure that you always use radians when dealing with trigonometric functions. Make sure your calculator is set to calculations in radians.

Case 2. cos(x) is NOT multiplication

$$\cos(x + y) \neq \cos x + \cos y$$
$$\cos(3x) \neq 3\cos x$$

In general, this example uses cosine, but it also applies to any of the six trigonometric functions, so be careful!

Case 3. Power of trigonometric functions

Remember that if *n* is a positive integer then

☑ $\sin^n x = (\sin x)^n$ ☒ $\sin^n x \neq \sin x^n$

The same holds for all the other trig functions as well of course. Also remember to keep the following straight.

$$\sin^2 x \ \text{ vs } \ \sin x^2$$

In the first case, we are taking the sine then squaring the result and in the second we are squaring the *x* then taking the sine.

Case 4. Inverse trigonometry notation

$$\cos^{-1} x \neq \frac{1}{\cos x}$$

In trigonometry, the -1 in $\cos^{-1} x$ is NOT an exponent, it is to denote the fact that we are dealing with an inverse trigonometric function. There is another notation for inverse trig functions, however, not commonly used.

$$\cos^{-1} x = \arccos x$$

Calculus Errors

Case 1. Derivatives and Integrals of Products/Quotients

Recall that

$$(f \pm g)'(x) = f'(x) \pm g'(x) \text{ and } \int f(x) \pm g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$$

	Derivatives	Integrals
Product	$(fg)'(x) \neq f'(x).g'(x)$	$\int f(x)g(x)\,dx \neq \int f(x)\,dx \int g(x)\,dx$
Quotient	$\left(\frac{f}{g}\right)'(x) \neq \frac{f'(x)}{g'(x)}$	$\int \frac{f(x)}{g(x)}\,dx \neq \frac{\int f(x)}{\int g(x)}$

Case 2. Proper use of the formula for $\int x^n\,dx$

Many students forget that there is a restriction on this integration formula, so for the record here is the formula along with the restriction.

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + c \text{ provided } n \neq -1.$$

Case 3. Dropping the absolute value when integrating $\int \frac{1}{x}dx$ **.**

The formula for $\int \frac{1}{x}dx = \ln|x| + c$, the absolute value bars on the argument are required! It is certainly true that on occasion they can be dropped after the integration is done, but they are required in most cases. For instance, contrast the two integrals,

$$\int \frac{2x}{x^2+10}dx = \ln|x^2+10| + c = \ln(x^2+10) + c$$

$$\int \frac{2x}{x^2-10}dx = \ln|x^2-10| + c$$

In the first case the x^2 is positive and adding 10 will not change the positive value since $x^2+10>0$, so, we can drop the absolute value bars. In the second case however, since we don't know what the value of *x* is, there is no way to know the sign of x^2-10 and so the absolute value bars are required.

Case 4. Improper use of the formula $\int \frac{1}{x}dx = \ln|x| + c$

In this case, students seem to make the mistake of assuming that if $\frac{1}{x}$ integrates to $\ln|x|$ then so must one over anything! The following table gives some examples of incorrect uses of this formula.

Integral	☒	☑
$\int \frac{1}{x^2+1}dx$	$\ln(x^2+1)+c$	$\tan^{-1}x+c$
$\int \frac{1}{x^2}dx$	$\ln(x^2)+c$	$-x^{-1}+c=-\frac{1}{x}+c$
$\int \frac{1}{\cos x}dx$	$\ln|\cos x|+c$	$\ln|\sec x+\tan x|+c$

So, be careful when attempting to use this formula. This formula can only be used when the integral is of the form $\int \frac{1}{x} dx$.

Case 5. Improper use of Integration formulas in general

This one is really the same issue as the previous one, but so many students have trouble with logarithms. For example, the general formula is:

$$\int \sqrt{u}\, du = \frac{2}{3} u^{3/2} + c \text{ or } \int u^2\, du = \frac{u^3}{3} + c$$

The mistake here is to assume that if these are true then the following must also be true.

$$\int \sqrt{\text{anything}}\, du = \frac{2}{3}\left(\sqrt{\text{anything}}\right)^{3/2} + c \text{ or } \int (\text{anything})^2\, du = \frac{1}{3}(\text{anything})^3 + c$$

This just isn't true! The first set of formulas work because it is the square root of a single variable or a single variable squared. Here's another table with a couple of examples of these formulas not being used correctly.

Integral	☒	☑
$\int \sqrt{x^2+1}\, dx$	$\frac{2}{3}\left(x^2+1\right)^{3/2} + c$	$\frac{1}{2}\left(x\sqrt{x^2+1} + \ln\left\lvert x + \sqrt{x^2+1}\right\rvert\right) + c$
$\int \cos^2 x\, dx$	$\frac{1}{3}\cos^3 x + c$	$\frac{x}{2} + \frac{1}{4}\sin(2x) + c$

Case 6. Dropping limit notation

Students tend to get lazy and start dropping limit notation after the first step. For example, an incorrectly worked problem is

☒	☑
$\lim_{x\to 3} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{x-3} = x+3 = 6$	$\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3}(x+3) = 6$

In the wrong (☒) section, the following mistake is listed :

Mistakes #1: $\lim_{x\to 3}\frac{x^2-9}{x-3}=\frac{(x-3)(x+3)}{x-3}$.

You're saying that the value of the limit is $\frac{(x-3)(x+3)}{x-3}$ and this is clearly not the case.

Mistakes #2: $x+3=6$

You are making the claim that each side is the same, but this is only true provided $x = 3$ and what you really are trying to say is $\lim_{x\to 3} x+3=6$.

Case 7. Improper derivative notation

Often time, the differentiation of $f(x)=x(x^3-2)$ is written as

$$f(x)=x(x^3-2)=x^4-2x=4x^3-2$$

The proper notation should be written as $f'(x)=...$

$$f(x)=x(x^3-2)=x^4-2x$$
$$f'(x)=4x^3-2$$

Case 8. Loss of integration notation

<u>*Example 1*</u>.

There are many dropped notation errors that occur with integrals. One of the examples is: $\int x(3x-2)\,dx=3x^2-2x=x^3-x^2+c$

The proper notation should be written as $\int x(3x-2)\,dx=\int\left(3x^2-2x\right)dx=x^3-x^2+c$

<u>*Example 2*</u>.

Another big problem in dropped notation is students dropping the *dx* at the end of the integrals. For instance, $\int x(3x-2)\,dx$.

<u>*Example 3*</u>.

Another dropped notation error that is also common is with the definite integrals. Students tend to drop the limits of integration after the first step and do the rest of the problem with implied limits of integration as follows :

☒ $\int_1^2 x(3x-2)\,dx = \int (3x^2-2x)\,dx = x^3 - x^2 = 8-4-(1-1) = 4$

☑ $\int_1^2 x(3x-2)\,dx = \int_1^2 (3x^2-2x)\,dx = \left(x^3 - x^2\right)\Big|_1^2 = 8-4-(1-1) = 4$

Case 9. Dropped constant of integration

Dropping the constant of integration on indefinite integrals (the + *c* part) is one of the biggest errors that students make in integration. There are actually two errors here that students make. Some students just don't put it in at all, and others drop it from intermediate steps and then just tack it onto the final answer.

Case 10. Misconceptions about $\frac{1}{0}$ and $\frac{1}{\infty}$.

This is not so much about an actual error that students make, but instead a misconception that can, on occasion, lead to errors. This is also a misconception that is often encouraged by laziness on the part of the instructor.

Often, we will write $\frac{1}{0} = \infty$ and $\frac{1}{\infty} = 0$. The problem is that neither of these are technically correct and in fact the second, depending on the situation, can actually be $\frac{1}{0} = -\infty$. All three of these are really limits and we just short hand them. What we really should write are $\lim_{x\to\infty} \frac{1}{x} = 0$; $\lim_{x\to 0^+} \frac{1}{x} = \infty$ and $\lim_{x\to 0^-} \frac{1}{x} = -\infty$.

In the first case 1 over something increasingly large is increasingly small and so in the limit we get zero. In the last two cases note that we've got to use one-sided limits as $\lim_{x \to 0} \frac{1}{x}$ doesn't even exist! In these two cases, 1 over something increasingly small is increasingly large and will have the sign of the denominator and so in the limit it goes to either ∞ or $-\infty$.

Case 11. Indeterminate forms

This is actually a generalization of the previous topic. The two operations above, $\infty - \infty$ and $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ are called indeterminate forms because there is no one single value for them. Depending on the situation they have a very wide range of possible answers. There are many more indeterminate forms that you need to look out for. As with the previous discussion there is no way to determine their value without taking the situation into consideration. Here are a few of the more common indeterminate forms.

$$\infty - \infty \qquad \frac{\infty}{\infty} \quad \frac{0}{0} \quad 0.\infty \quad 0^0 \quad 1^{\infty} \quad \infty^0$$

Let's just take a brief look at 0^0 to see the potential problems. Here we really have two separate rules that are at odds with each other. Typically, we have $0^n = 0$ (provided n is positive) and $a^0 = 1$. Each of these rules implies that we could get different answers. Depending on the situation we could get either 0 or 1 as an answer here. In fact, it's also possible to get something totally different from 0 or 1 as an answer here as well. All the others listed here have similar problems. So, when dealing with indeterminate forms you need to be careful and not jump to conclusions about the value.

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